GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

BER PERFORMANCE ANALYSIS OF GROUPED ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING INTERLEAVE DIVISION MULTIPLE ACCESS

Mr. Aravind Chetti [*1] and Mrs. Ch. Pratyusha Chowdary[2]
[*1]Student, GRIET College
[2]Assistant Professor ,GRIET College

ABSTRACT

A generalized version of orthogonal frequency division multiplexing interleave division multiple access (OFDMIDMA) referred to as grouped OFDM-IDMA (G-OFDM-IDMA)is introduced in this paper. By dividing users into groups and transmitting each group's data only on some (as opposed to all) subcarriers, G-OFDM-IDMA can have much lower decoding complexity compared with conventional OFDM-IDMA while preserving the bit error probability (BEP) performance and the bandwidth efficiency. The user grouping problem is formulated into an integer linear programming problem whose suboptimal solution is proposed and compared with the lower bound. The optimization complexity issue is also addressed. Simulations are carried out to test the performance of G-OFDM-IDMA under various system configurations. It is observed that up to 80% complexity could be saved when the conventional OFDM-IDMA is substituted by G-OFDM-IDMA configured according to the suboptimal grouping solution.

Keywords— *OFDM-IDMA, user grouping, linear programming, complexity reduction, diversity order.*

I. INTRODUCTION

The interleave division multiple access (IDMA) system has drawn an increasing research interest over recent years [1]–[5]. Two key features, namely, high power efficiency and low decoding complexity, make IDMA a promising candidate among several multiple access schemes for future wireless communications. The high power efficiency is an inherent property of non-orthogonal multi-user transmission and is typically realized by power optimization among users [6]. As a result, power optimization of IDMA has been extensively studied in the literature (see, e.g., [7]–[11]). In this paper, however, we will focus on the complexity aspect of such systems.

For multiple access systems, simple single user detectors such as matched filter may not work well due to the multi-ple access interference (MAI) introduced by other users. To achieve satisfactory performance, more complex multi-user detectors (MUD) that jointly decode all users data are required at the receiver. For the well-known code division multiple access (CDMA) system, its decoding complexity is usually high when MUD is utilized. For example, the minimum mean square error (MMSE) based MUD for CDMA has per-user complexity that is quadratic in the number of users [12]. On the contrary, by utilizing turbo-type MUD to remove the MAI, the per-user decoding complexity of IDMA is independent of the number of users [1], [3], [13]. This feature is very appealing for system realization. In multipath fading channels, the decoding complexity of IDMA also increases linearly with the number of channel taps [1], [13]. In such cases, orthogonal frequency division multiplexing (OFDM) can be incorporated into the IDMA system to resolve the multipath effects by converting frequency selective channels into parallel flat fading sub channels. The resultant combined system, referred to as OFDM-IDMA [14]–[16], will entail decoding complexity equivalent to that of IDMA in flat fading channels regardless of the number of channel taps. As a consequence, OFDM-IDMA is more suitable for multipath channels than plain IDMA. However, when the number of users is large, there would still be a significant computation cost for the receiver, as multiple iterations are needed in OFDM-IDMA. To further ease the computation, we recognize that the decoding complexity of OFDM-IDMA is actually linear in the number of users sharing a particular sub-carrier rather than the total number of users of the system. Although these two numbers are identical for conventional OFDM-IDMA as every sub-carrier is shared by all users, it will not be the case if each user's data is only transmitted on a subset of the subcarriers rather than all of them. This is the motivation for our proposed Grouped OFDM-IDMA, or G-OFDM-IDMA, in this paper. In G-OFDM-IDMA, users and subcarriers are divided into a number of groups, and each user group's data is only transmitted on the corresponding group of subcarriers. We will show that this design is capable of further reducing the complexity while preserving the bit error probability (BEP) performance and bandwidth efficiency of conventional OFDM-IDMA.

Several papers have investigated the issue of user grouping for IDMA based systems. In [17], user grouping for plain IDMA is proposed to reduce the severity of MAI by allocating orthogonal spreading codes to different user groups. However, only the performance in additive white Gaussian noise (AWGN) channels is considered therein. For multipath channels, the inter-group interference is still present after de-spreading and the BEP performance will degrade. In [18], user grouping for OFDM-IDMA is proposed and claimed to be capable of improving the spectral efficiency. However, a grouping method based on instantaneous channel response is mentioned without providing further details. In [19], we introduced G-OFDM-IDMA from the complexity reduction perspective. In that paper, no grouping algorithm is given but simple uniform grouping is employed to illustrate the potential benefit of G-OFDM-IDMA. In this paper, we extend the idea in [19] and propose a rigorous grouping algorithm that is applicable to more general scenarios with arbitrary number of users. Not only nonuniform group size but also nonuniform power allocation will be considered in this paper. The user grouping issue is formulated into an integer linear programming (ILP) optimization problem. A suboptimal solution is given and its performance is compared with both the lower bound and the conventional case. The optimization complexity issue is also addressed and simulations are carried out to show the validity of the proposed grouping method.

The rest of this paper is organized as follows. Section II briefly reviews the conventional OFDM-IDMA system. Section III introduces the G-OFDM-IDMA concept. Section IV presents our grouping algorithm. Simulations are shown in Section V and conclusions are drawn in Section VI.

Notation: In denotes a size-n identity matrix. La(·), Lp(·) and Le(·) denote the a priori, the a posteriori and the extrinsic log-likelihood ratios (LLRs), respectively. O(f(n)) is defined as a function whose magnitude is upper-bounded by a constant times f(n), for all large n [20]. x (x) is the smallest (largest) integer that is equal to or greater (less) than x. minm,n picks the smaller between m and n.

II. CONVENTIONAL OFDM-IDMA

We first briefly review the up-link transmission of a conventional OFDM-IDMA system with a total of K users and N subcarriers. Fig. 1 shows the discrete-time block diagram of an OFDM-IDMA transmitter for user-k and the receiver. At the transmitter the information bits uk are first spread by a length-S spreading sequence, which can be regarded as repetition coding. The data sequence after spreading, denoted by ck, is referred to as chips. The chips are then interleaved by a user-specific interleaver πk, which serves as the only means of separating users, thus giving the name interleave division multiple access (IDMA). The interleaved chips c˜ are modulated using quadrature phase shift keying (QPSK), giving rise to the modulated symbols Xk which are modulated onto the N subcarriers via inverse discrete Fourier transform (IDFT).

After inserting the cyclic prefix (CP), the time domain OFDM symbol are transmitted through an L-path fading channel. At the transmitter, only the average channel gain rather than the instantaneous channel state information(CSI) is known. Without loss of generality, a Rayleigh fading channel model with correlation matrix IL/L is assumed for all users, where IL is a size-L identity matrix.

At the receiver, CSI is assumed perfectly known. A special MUD referred to as elementary signal estimator (ESE) is adopted, which essentially performs chip-level soft interfer-ence cancellation (SIC) in frequency domain on a per subcar-rier basis [1], [2]. The input of ESE consists of the frequency domain received signal Y and La(c˜)s. La(c˜)s are given by the soft-input soft-output decoders (SISO DECs) at the previous iteration and used to obtain the soft estimates of the transmitted signals for SIC. The output of ESE, Le(c˜)s, are enhanced soft information on c˜ after SIC. The de-interleaved

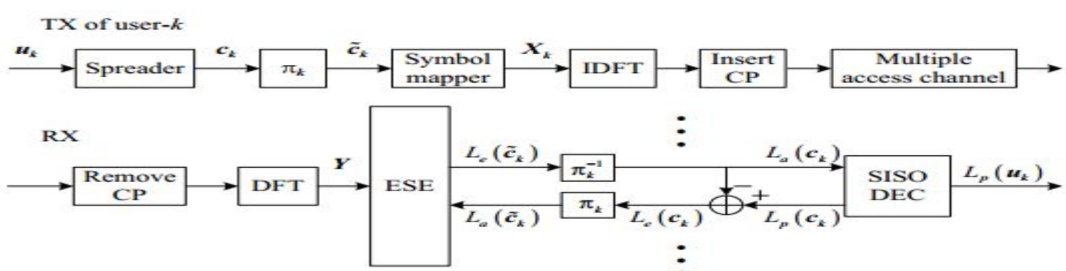

Fig. 1. Transmitter of user-k and receiver in the uplink of an OFDM-IDMA system.

(via $\pi-1$) version of Le(c˜)s are regarded as La(ck)s and fed into the corresponding SISO DECs. According to the standard a posteriori probability (APP) decoding [21], [22], the DEC computes Lp(ck) based on La(ck) and the code structure. For repetition code, the DEC simply de-spreads La(ck) to generate Lp(uk) and then spreads Lp(uk) again to obtain Lp(ck). After subtracting La(ck) from Lp(ck), the resultant Le(ck) are interleaved again and fed back into ESE. The ESE and DECs perform the above procedure iteratively and in the final iteration the DEC for user-k only computes Lp(uk). The information bits are then estimated according to the signs of Lp(uk). A more detailed description of this procedure can be found in [23].

Data rate and bandwidth efficiency: Let Ts denote the sampling interval, then the data rate of each user and the bandwidth **efficiency** of the system are given by

Rb = NTS = STs , η = 1/Ts = S . (1)

Decoding complexity: An attractive feature of OFDM-IDMA is its low decoding complexity; that is, the complexity per user per iteration is independent of both the number of users K and the number of multipaths L [15], [24]. In addition, as ESE performs SIC on every subcarrier, the total complexity is linear with N. In summary, the total decoding complexity per OFDM symbol is O(KNQ) where Q is the number of iterations. The exact expression of the decoding complexity in terms of the number of different operations can be found in [24], which shows that the complexity is almost strictly linear with KNQ, as the complexity of DFT and spreading/de-spreading operations are negligible.

III. PROPOSED G-OFDM -IDMA

As ESE essentially performs SIC, the complexity of OFDMIDMA per iteration per subcarrier is actually linear with the number of users sharing that particular subcarrier rather than
the total number of users of the system. It is thus possible to further reduce the complexity by restricting the number of users sharing the same subcarrier, which is particularly beneficial if K and Q are both large. This motivates us to generalize the OFDM-IDMA scheme and propose the G-OFDM-IDMA concept, where users and subcarriers are
partitioned into several groups and each subcarrier group is allocated exclusively to a user group. By doing so we expect to reduce the total decoding complexity while maintaining the overall system performance. Moreover, via grouping, we can DANG et al.: OFDM-IDMA WITH USER GROUPING 1949 also reduce the buffer size for interleaving, since the spreading
length is reduced accordingly, as will be detailed later.

A. Transmitter and Receiver Structures

We keep the total number of users K and number of subcarriers N unchanged as in conventional OFDM-IDMA systems. In G-OFDM-IDMA, however, the users and subcarriers are both divided into G groups with Kg users in the g-th user group and Ng subcarriers in the g-th subcarrier group, respectively, where Ng is chosen to be proportional to Kg ,
i.e., Ng /N = Kg /K . The users in group-g transmit their data only on subcarriers of group-g. Fig. 2 shows the transmitter structure of a G-OFDM-IDMA system for user-k in group-g and the receiver structure. The superscript (g) appearing in the figure denotes the group index. For any particular group of users, the transmitter and receiver diagrams are nearly the same as those of the conventional OFDM-IDMA system. The only difference is that, instead of all N subcarriers, only a subset of subcarriers are utilized for each user. To this end, a subcarrier allocation module is inserted as shown in Fig.2.In this paper, the subcarriers among different groups are assumed to be allocated in an interleaved fashion since the transmitter does not know CSI. It is worth noting that when G =1 there is only one group and G-OFDM-IDMA reduces to the conventional OFDM-IDMA. When G = K,thereis
only one user in each group and the G-OFDM-IDMA system boils down to an orthogonal frequency division multiple access (OFDMA) system with spreading and interleaving. In this
sense, OFDM-IDMA and OFDMA can be viewed as the two extremes of G-OFDM-IDMA in terms of the number of groups.

B. Data Rate and Spreading Length Selection

The data rate of users in group-g and the system bandwidth
efficiency are given by

$$R_b^{(g)} = \frac{2N_g/S_g}{NT_s} = \frac{2N_g}{S_gNT_s}, \ \eta_G = T_s\sum_{g=1}^{G} K_g R_b^{(g)}, \qquad (2)$$

where Sg is the common spreading length for users in groupg and the sampling interval Ts is identical to that in OFDMIDMA. It is natural to use a shorter spreading length for each group. Since our focus is on complexity reduction, we assume for fairness that the spreading length is chosen such that each user's data rate is not altered by grouping, i.e., $R_b^{(g)} = R_b$

$$\frac{S_g}{S} = \frac{N_g}{N} = \frac{K_g}{K}, \ g = 1, 2, \ldots, G. \qquad (3)$$

In other words, Sg is reduced according to K_g .As K_g , N_g and S_g are all integers, the constraint of (3) may not be strictly satisfied. Instead, the following approximations are employed in practice. S_g is set to be [K_g S/K], while N_g is set to be [K_g N/K].

C. Decoding Complexity

Based on the analysis in Section II, one can easily obtain the decoding complexity of G-OFDM-IDMA as

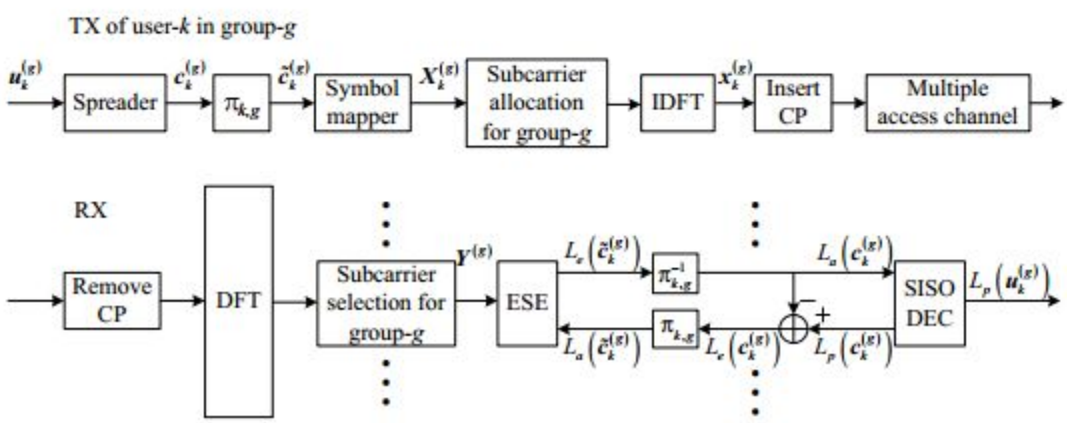

Fig. 2. Transmitter of user-k in group-g and receiver in the uplink of a G-OFDM-IDMA system.

$\mathcal{O}(\sum_{g=1}^{G} K_g N_g Q_g)$, where Q_g is the number of iterations for group-g. With the relationship in (3), we have $\sum_{g=1}^{G} K_g N_g Q_g = \frac{N}{K}\sum_{g=1}^{G} K_g^2 Q_g$, which implies that the group size$\{K_g\}_{g=1}^{G}$ has an important effect on the complexity. Roughly speaking, smaller K_g results in lower complexity.

D. Diversity Order

Besides the bandwidth efficiency, the BEP performance should also be preserved for a fair complexity comparison between G-OFDM-IDMA and OFDM-IDMA. While it is difficult to obtain a closed-form BEP expression, one can study the diversity order instead, which characterizes the BEP behavior at high signal-to-noise ratio (SNR) and unveils the decisive factors of the BEP. The result can serve as a guidance when choosing parameters for G-OFDM-IDMA system. First, we analyze the diversity order of conventional OFDMIDMA. When the iterative MUD at the receiver converges, the MAI for each user diminishes to zero, leading to a BEP performance approaching that of a single-user case. Therefore, the maximum diversity order can be derived based on a single user system, i.e., an OFDMA system with spreading and interleaving. In [25], it is proved that the maximum achievable diversity order for an OFDMA system with repetition coding is min{S, L} if the channel is of full rank (which is assumed in this paper). Therefore, min{S, L} is an upper bound of the diversity order of an OFDM-IDMA system. This result can be applied to G-OFDM-IDMA on a per group basis. In other words, the diversity order of users in group-g, Dg , is upper bounded by

Dg ≤ min{Sg ,L}, (4)

where the upper bound can be achieved if the MAI for each user is completely mitigated. This result reveals the important role of the spreading length, or equivalently, the group size as a consequence of (3), on the BEP performance. Generally speaking, a spreading length which is equal to or a bit larger than the channel length would be adequate to fully exploit the frequency diversity.

IV. GROUPING ALGORITHM

Analysis in Section III reveals the potential of complexity reduction by user grouping without sacrificing performance. However, it does not provide a grouping algorithm. The diversity order only characterizes BEP at high SNR and after sufficient number of iterations to cancel the MAI. In practical scenarios, however, the operating SNR may not be that high, and the bit error rate (BER) requirement also varies with different applications. Moreover, though shorter Sg means smaller Kg and correspondingly lower decoding complexity, the needed number of iterations, Qg , is observed by simulations likely to increase as Sg decreases for a fixed ratio of Kg /S g , counteracting the benefit of reduced complexity. Therefore, a grouping algorithm needs to be designed in order to realize the benefits of G-OFDM-IDMA in practical systems. In this section we formulate the grouping problem into an ILP one and give a suboptimal solution as well as a lower bound. A. Problem Formulation For a system with K users and spreading length S,define p 0 (0 <p0 < 0.5) as the target BER for each user and Γ0 as the SNR per bit averaged across all users. Then given Kg (Sg)and p 0, the needed number of iterations Qg ,which is the minimum number of iterations to achieve p 0 for users in group-g at a specific average SNR per bit γ g , can be obtained by SNR-variance evolution. SNR-variance evolution is a semianalytical BER assessment method. It tracks the average SNR and variance of the symbols at the output of the ESE module for each iteration and evaluates the BER performance at the final iteration based on the latest updated SNR when applying to IDMA-based systems. It was originally proposed for plain IDMA systems [2], [26] and later extended to OFDM-IDMA systems by using the average channel gains [23], [27]. Therefore, the predicted BER performance of OFDM-IDMA is the averaged BER over all channel realizations rather than the instantaneous channel realization. Here we apply this method to each user group in a G-OFDM-IDMA system.

With the available Qg and according to Sections II and III-C, the decoding complexity for the group with Kg users is almost strictly linear with Ng Kg Qg = K 2 g Qg N/K.Asthe constant N/K does not affect the final result, we simply drop it and denote K 2 g Qg as the normalized decoding complexity that will be used subsequently. A 3-tuple (Kg ,γ g ,c g) is sufficient to characterize a group with Kg users at SNR γ g , where c g is defined as the per user decoding complexity given by c g = Kg Qg . Our goal is to find the optimal user grouping profile, defined as the set {G;(K1 ,γ 1 ,c1),..., (KG,γG,cG)}, that minimizes

$$\sum_{g=1}^{G} K_g c_g, \text{ subject to } \sum_{g=1}^{G} K_g = K \text{ and } \sum_{g=1}^{G} K_g \gamma_g = K\Gamma_0.$$

Since we do not require γ g to be the same as Γ0,power allocation is also involved in the grouping procedure. However, as γ g is continuous, this optimization problem may have an infinite number of optimal solutions. Intuitively, for any optimal grouping profile, one can transfer some power from one group to another and the amount of the transferred power is so small that the number of iterations of the two groups do not change, leading to a new optimal profile. To avoid this ambiguity, one needs to constrain γ g within a limited set. Note that Qg (and c g) is a non-increasing staircase function of γ g , i.e., each Qg corresponds to an interval of SNR. Therefore, it would suffice to just use the beginning points of those intervals. Correspondingly, for a group with m users, only limited number of 3-tuples (m, γm,i,c m,i), i =1, 2,...,Im, survive and they are referred to as primitive tuples. The number of primitive tuples Im can be determined

by the following rules. An upper bound of the possible average SNR is given by KΓ0/m. This value determines the smallest number of iterations. A lower bound of the average SNR is
given by the smallest SNR that can achieve the target BER in a single user system and can be obtained directly from the BER-SNR curve (the g(·) curve in [26] and [2] for SNRvariance evolution). This lower bound is used to determine the upper bound of the number of iterations. In addition, the maximum number of iterations is constrained to be less than
KcK,1/m 2 ,where c K,1 is the per user complexity of the conventional system. Note that when m = K, only a single primitive tuple exists. These conditions are then sufficient
for the determination of both the number and values of the primitive tuples for any m, 1 ≤ m ≤ K.When m =1, there exists no more than one primitive tuple as the number of iteration is 1.

Consider a system in a 6-path channel with K =47 users, spreading length S =32, Γ0 =16 dB and p 0 =10 −4 as an example. Fig. 3 shows the related concepts during the SNR discretization. Each staircase curve corresponds to the function of the per user complexity versus the average SNR for a certain m and all markers on that curve denote the primitive tuples after SNR discretization. For example, only 7 primitive tuples survive for m =5, contrasting to the infinite 3-tuples on each point of the curve without discretization. Fig. 3 only shows the results for $1 \leq m \leq 8$ for clarity purpose. Note that the lines corresponding to m =1, 2, and 4 do not exist because they are infeasible, i.e., for any of those m, even at the upper bound of the SNR and with KcK,1/m 2 iterations, the system still can not achieve the target BER.

Now the grouping problem has been reformulated as finding the optimal combination of the primitive tuples which is a standard ILP one as follows.

User grouping problem (ILP-1)

Find the set of coefficients $\mathcal{L} = \{l_1; l_{2,1}, \ldots, l_{2,I_2}; \ldots; l_{K-1,1}, \ldots, l_{K-1,I_{K-1}}; l_{K,1}\}$ *that minimizes*

$$C = \sum_{m=1}^{K} \sum_{i=1}^{I_m} l_{m,i} m c_{m,i},$$

subject to

$$\text{(ct.1)} \quad \sum_{m=1}^{K} \sum_{i=1}^{I_m} l_{m,i} m = K,$$

$$\text{(ct.2)} \quad \sum_{m=1}^{K} \sum_{i=1}^{I_m} l_{m,i} m \gamma_{m,i} \leq K\Gamma_0,$$

(ct.3) $l_{m,i}, m = 1, 2, \ldots, K,$ and $i = 1, 2, \ldots, I_m,$ for each m, are non-negative integers.

This optimization problem is referred to as ILP-1. Note that the constraint (ct.2) is changed from an equality constraint to an inequality one as a result of the SNR discretization. The optimal solution and the corresponding value of the objective function are denoted by L_{opt} and C_{opt}, respectively.

B. Sub-optimal Solution and Lower Bound

Many efficient methods can be used to solve this standard ILP problem and branch-and-bound method is employed in this paper (see, e.g., branch-and-bound and other methods summarized in [28]). However, the optimization complexity could be very high as the total number of primitive tuples is large, e.g., there are 231 primitive tuples in the example of Fig. 3. Therefore, it is not worthy of searching for the optimal solution but a sub-optimal solution with low optimization complexity is more appropriate. In this paper we propose the following sub-optimal solution. First, we obtain an upper bound of C_{opt} by considering a special case in which the group-wise average SNR is the same as Γ0. In the interpretation of ILP, we only consider a single primitive tuple (m, $\gamma_{m,i}$,$c_{m,i}$) for each possible m. This tuple is chosen such that $\gamma_{m,i}$ is the largest one that does not exceed Γ0. This new ILP problem is referred to as ILP_{eq}. Now there exist no more than K optimization variables and only (ct.1) and (ct.3) need to be satisfied. Therefore, ILP eq can be easily solved. The optimal solution of ILP eq is denoted by L_{eq} and the corresponding value of the objective function is denoted by C_{eq}.

L_{eq} and C_{eq} are important intermediate results. They are then utilized in the procedure of deriving the sub-optimal solution of ILP-1. First, as an upper bound of C_{opt}, C_{eq} can be compared with $C_{conv} = K_{cK,1}$ to provide an overview of the extent of complexity reduction with user grouping. In addition, C_{eq} is used to filter out the primitive tuples which satisfy $m_{cm,i} \geq C_{eq}$ as they are obviously not a feasible candidate in ILP-1. Those tuples are basically the ones with low SNRs. A large portion of the primitive tuples can be filtered out as indicated by simulation, e.g., in the example of Fig. 3, 151 tuples will be filtered out. However, the number of remaining tuples may still be large in some cases. More rigorous restrictions will be imposed to the remaining tuples in the following steps. Leq is used to filter out the tuples with $m>m^*$,where m^* is the largest group size in solution L_{eq}. Moreover, the maximum number of candidate tuples for each m is also restricted as follows. For each m,atmost ($n_L + n_R$) tuples with SNRs in the vicinity of Γ0 are retained; that is,at most nL tuples with SNRs closest to but no larger than Γ0 are retained and at most n_R tuples with SNRs closest to but larger than Γ0 are also retained. Empirical results suggest n_L=2 and n_R=1 achieve good complexity-performance trade off (see the filled markers in Fig. 3). This restriction is intuitively reasonable as the chosen lower and upper bounds of the SNR are rather loose. A similar scenario is studied in [29] and a reference therein, which concluded that only a few power levels are sufficient to obtain good performance. As a consequence of the restriction, only a very limited number of primitive tuples need to be generated. At this stage, ILP-1 has been reduced to a new ILP problem with a much smaller scale. The new problem is referred to as ILP_{app} and its optimal solution and corresponding objective function value are denoted by L_{app} and C_{app}, respectively. Lapp is used as an approximation of L_{opt}. Clearly, C_{app} is a tighter upper bound of C_{opt} compared with C_{eq}. Lapp is only a sub-optimal solution of ILP-1.To make a more thorough performance assessment of this sub-optimal solution, it is desirable to compare C_{app} with a lower bound of C_{opt}. A natural way to obtain a lower bound of C_{opt} isto transform ILP-1 into its relaxation form; that is, an LP problem without the integer constraints ((ct.3) in ILP-1). This

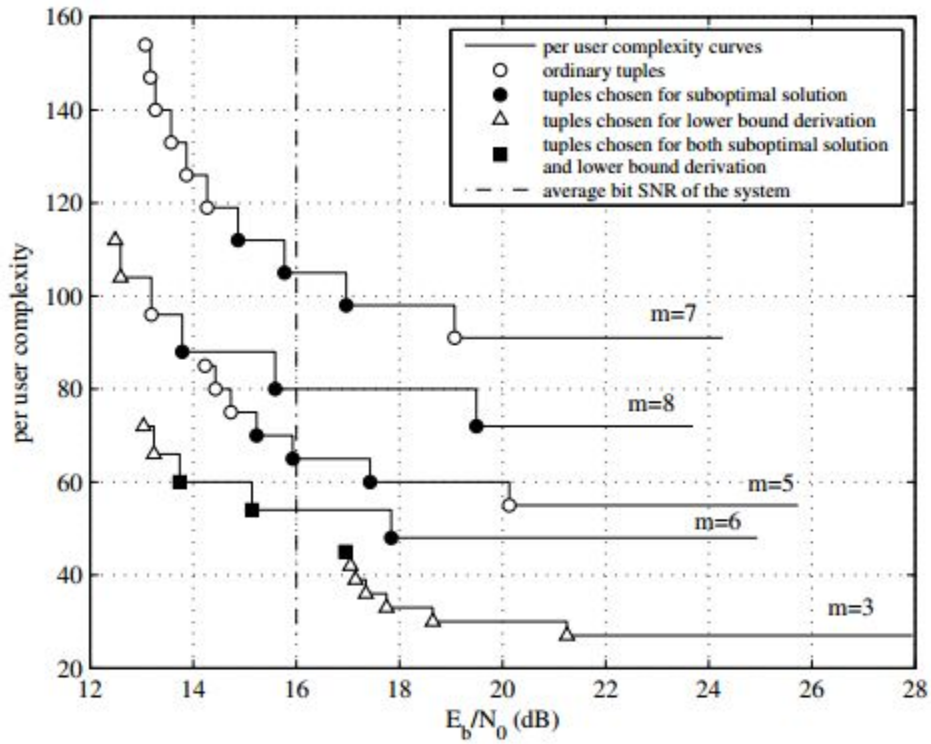

Fig. 3. Illustration of tuples in grouping optimization with K =47,S =32, Γ0 =16 dB and p0 =10−4

LP problem is referred to as **LP-1** and its optimal objective function value, C_{lb}, is a lower bound of C_{opt}. To solve **LP-1**, the number of tuples should be reduced according to the following rules. For any tuple $(m_1,\gamma_{m1,i1},c_{m1,i1})$, if there exists another tuple $(m_2,\gamma_{m2,i2},c_{m2,i2})$ such that $\gamma_{m2,i2}\leq \gamma_{m1,i1}$ and $c_{m2,i2}\leq c_{m1,i1}$ then $(m_1,\gamma_{m1,i1},c_{m1,i1})$ can be safely removed from LP-1 without alternating its optimal solution.The reason is that $(m_1,\gamma_{m1,i1},c_{m1,i1})$ can be substituted by m_1/m_2 times of $(m_2,\gamma_{m2,i2},c_{m2,i2})$ without increasing neither the power consumption nor the complexity. As a consequence,in Fig. 3, only the tuples on the left and bottom "borders" (see the square and triangle markers) of the plain are kept for LP-1 and this problem can be quickly solved. The gap between C_{app} and C_{lb} will be illustrated by simulations in Section V.

C. Complexity Analysis and User-Wise Power Allocation

SNR-variance evolution is a semi-analytical method. The variance-SNR and BER-SNR curves are obtained by simulating a single user system. This is the main computation cost in SNR-variance evolution. With the two curves available, the evolution itself is very simple and its complexity is negligible. In G-OFDM-IDMA, one needs to simulate multiple curves for different spreading lengths, which will be the main computation cost of the grouping procedure. Fortunately, those curves can be stored and reused for different K, Γ0 ,and P_0 values, as well as systems with spreading length that is smaller than S. In addition, closed-form expressions of the curves exist for repetition codes in some special scenarios. For example, Analytical representations are derived for AWGN channels in [30]. The results can be readily extended to typical fading channels such as Rayleigh fading channels with the assumption that the fading coefficients are uncorrelated across subcarriers. Denoting ρ as the squared sum of S sub carrier coefficients (those subcarriers correspond to a same bit after spreading and interleaving), [30] showed that the BER-SNR (i.e., BER-γ s) curve also depends on the constant ρ = S in the AWGN channel. Assuming uncorrelated Rayleigh fading channels, ρ can be shown to be a chi-squared random variable

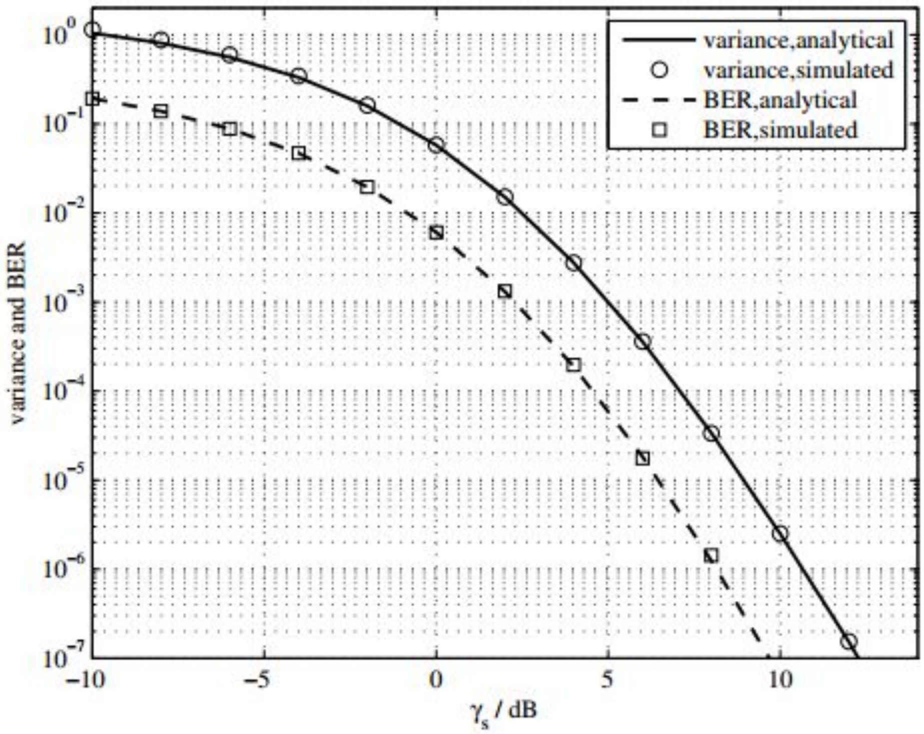

Fig. 4. Comparison of analytical and simulated variance-SNR and BER-SNR curves for SNR-variance evolution in uncorrelated Rayleigh fading channel with S =8.

with 2S degrees of freedom [19], [31]. The BER-SNR curve can then be obtained by averaging the result in AWGN channel over the distribution of ρ, which can be numerically evaluated.

The variance-SNR curve can be obtained in a similar manner.Fig. 4 compares the evaluation of the analytical expressions with simulation results in uncorrelated fading channels for

S =8 and one can find that they match quite well. The model of uncorrelated fading among subcarriers is also assumed in [15], [23] and will be used later in simulations in this

paper. Note that in practice, this model is actually applicable even in correlated channels (but should not be static-flat fading channels) provided that the chips corresponding to a

single bit are uncorrelated, which is approximately true if deep interleaving (e.g., interleaving spans multiple OFDM symbols) is employed. Now the curves needed in the grouping algorithm can be obtained through numerical evaluations that is much faster than pure simulations. Finally, the optimization of ILP_{app} is rather simple since the number of tuples is reduced and kept to a minimum.

In the grouping procedure, we only addressed the group wise power allocation. For each group, user-wise power allocation should also be considered, which inevitably increases

the optimization complexity. However, from [7], [8], [11] one can find that the user-wise power allocation result is in fact independent of the average SNR; that is, the optimal power distribution among users is fixed for any given number of users, spreading length and p0. This is a good property as one only needs to perform user-wise power allocation (the result is a power distribution among users) once for each group size (rather than for each tuple) beforehand and the remaining user grouping optimization procedure is still the same as described in Sections IV-A and IV-B. Moreover, the needed varianceSNR and BER-SNR curves for user-wise power allocation are already available.

V. SIMULATIONS

In this section, we provide simulations to illustrate the viability of the proposed G-OFDM-IDMA scheme.

TABLE I
USER GROUPING PROFILES OF A SYSTEM WITH K =47, S =32 AND p0 =10 −4 OPERATING AT Γ0 = 16dB IN MULTIPATH RAYLEIGH FADING CHANNELS

system	problem	grouping result (unit of SNR: dB)	decoding complexity	simulated BER
grouped OFDM-IDMA	$\mathrm{ILP_{eq}}$	1×[5, 15.9313, 65] 7×[6, 15.1395, 54]	C_{eq}=2593	/
	$\mathrm{ILP_{app}}$	8×[3, 16.9498, 45] 1×[5, 15.9313, 65] 2×[6, 13.7395, 60] 1×[6, 15.1395, 54]	C_{app}=2449	0.52×10^{-4}
	ILP-1	4×[3, 17.1498, 39] 4×[3, 17.3498, 36] 1×[5, 15.2313, 70] 3×[6, 13.7395, 60]	C_{opt}=2330	/
	LP-1	8.25× [3, 17.3498, 36] 3.71× [6, 13.7395, 60]	C_{lb}=2225.78	/
OFDM-IDMA	/	1×[47, 16, 376]	C_{conv}=17672	1.13×10^{-7}

A. BER and Complexity Tradeoff

First, we present in Table I the final grouping results of different problems in the example of Fig. 3 as described in IV-B. It can be seen that different problem formulations result in different grouping profiles. As for the complexity, C_{app} is very close to the optimal solution C_{opt} and much lower than C_{conv} , showing the effectiveness of the proposed grouping algorithm. Note that the simulated BER (after 8 iterations) of the conventional OFDM-IDMA is far smaller than the target BER in this example, as opposed to the BER of GOFDM-IDMA that is similar to the target BER. One may wonder whether the required number of iterations for OFDMIDMA could be overestimated by the SNR-variance evolution. However, we found through brute-force simulations that 8 is indeed the minimal number of iterations satisfying the preset target BER. If, for example, even one less iteration will result in a BER exceeding the preset target BER value.

Next, we turn to more extensive simulations. Consider a system with total user number K =32, subcarrier number N = 512, and bandwidth efficiency ηG =2 bits/s/Hz (i.e.,S =32). An L =3 paths Rayleigh fading channel model (with identity correlation matrix) is assumed for all users. We evenly divide the users into 1, 2, 4, 8, 16, 32 groups with Kg =32, 16, 8, 4, 2, 1 users in each group, respectively. Thus for each group, Sg = Kg . Note that Kg =32 corresponds to the conventional OFDM-IDMA system and Kg =1 leads to an OFDMA system. The BER performance and the corresponding decoding complexity for different group sizes at different SNR values are simulated with 10 iterations. Fig.5 plots the average BER performance over 5000 channel realizations. The complexity are indicated by the variable C in the legends. Fig. 5 shows the possible benefits of user grouping, i.e., reducing the decoding complexity without compromising the

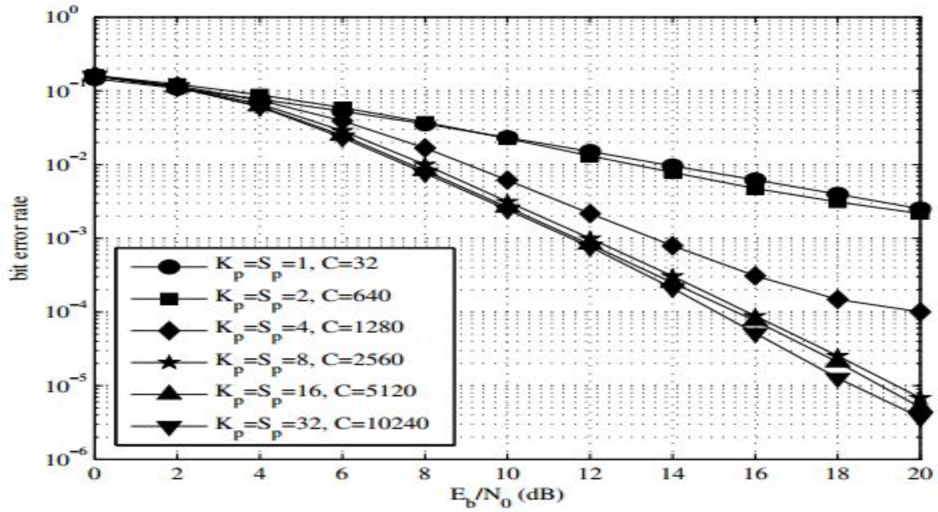

Fig. 5. BER and complexity of G-OFDM-IDMA with different group sizes for channel length L =3

BER performance. One can observe that for Kg =8, 16 and 32, the diversity orders (the absolute value of the slope of the BER curve in dB/dB scale at high SNR range [32]) are the same. Increasing the spreading length would double the decoding complexity but the SNR gains are only marginal. Therefore, grouping the users with a shorter spreading length is desirable. However, too short spreading length compared with the number of channels taps, though further reducing the complexity, results in smaller diversity order and thus degraded BER performance, as shown by the BER curves of Kg =2 and 1. Intuitively, a medium spreading length is preferable. Note that the full diversity order (min{Sg ,L})is not achieved for Kg =2 and 4, indicating the performance does not converge to a single user case. This is consistent with the conclusion that min{Sg ,L} is only an upper bound of the diversity order. Therefore, the proposed grouping algorithm with more careful manipulations in Section IV is needed to guarantee satisfactory user grouping.

B. Complexity Comparison

In this subsection we compare the decoding complexity using the proposed grouping algorithm under different system configurations. Specifically, the spreading length is fixed to be S =32 but the number of active users is changing in the range K =8 to K =56. Four target BERs, 10 −3 , 10 −4 , 10 −5 and 10 −6 , are tested at average SNRs Γ0 =10 dB, Γ0 =15 dB and Γ0 =25 dB, respectively. Independent fading among subcarriers is assumed thus the needed BER-SNR and varianceSNR curves can be obtained in analytical form. Fig. 6 plots the complexity ratios, i.e., C_{app}/C_{conv} and C_{lb}/C_{conv} with respect to the number of users for different target BERs at Γ0 =10 dB. Fig. 7 and 8 show the similar curves at Γ0 =15 dB and Γ0 =25 dB, respectively.

Several facts can be observed from Figs. 6 - 8. First, all the curves again verify the capability of complexity reduction through user grouping, e.g., C_{app} is only a few percent of C_{conv} in many cases. Second, the proposed user grouping algorithm (ILP_{app}) performs quite well and its performance

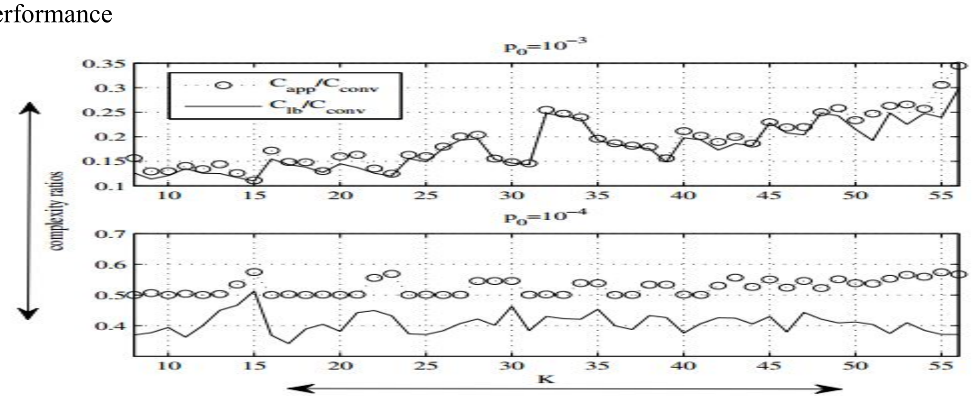

Fig. 6. Complexity ratios versus number of users at Γ0 =10 dB.

is very close to the lower bound, thus close to the optimal performance. In some scenarios such as in the sub figure of Fig. 8 with $p_0=10^{-3}$, the performance of the proposed algorithm achieves the lower bound, indicating that it is in fact optimal. Third, a jump of the complexity ratio around K =32 could be observed in Fig. 8 with $p_0 = 10^{-4}$, 10^{-5} and 10^{-6} .This phenomenon arises due to the rounding of the group wise spreading length given by S_g = K_g S/K . In general, for a given K_g, S_g is larger when K<S =32 compared to K ≥ S =32, leading to candidate tuples with lower complexities. As a consequence, a jump of the complexity ratio after grouping optimization may appear around K = S.

Fourth, by investigating the single user performance which is the lower bound of the multi-user case for the given configurations, we find that the minimal required Γ0 for BER of 10^{-3} is about 7.2 dB. Therefore, though the grouping algorithm is applicable in any SNR region, its result is meaningful only when the operating SNR is higher than 7.2 dB. In this sense, the chosen SNRs in Figs. 6 - 8 can be interpreted as the low, medium and high SNRs, respectively. For the low SNR of Γ0 =10 dB, the target BERs of 10^{-5} and 10^{-6} can not be achieved any more even without grouping. One can conclude that user grouping is more beneficial with higher target BER at higher SNR. This can be intuitively explained as the available channel capacity is higher in those cases. The high channel capacity is exploited through user grouping to reduce the complexity in this paper. Alternatively, the channel capacity can be exploited to maximize the data rate. In fact, rate optimization of equal power IDMA system in AWGN channels has been reported in [33], where users are also divided into multiple groups according to their rates. Interestingly, a potential extension of the framework in this paper is to perform power minimization with complexity constraint, e.g., the per user decoding complexity is restricted to be lower than a certain threshold. As user grouping creates an additional optimization dimension, improved performance is expected compared with the non-grouping scenario.

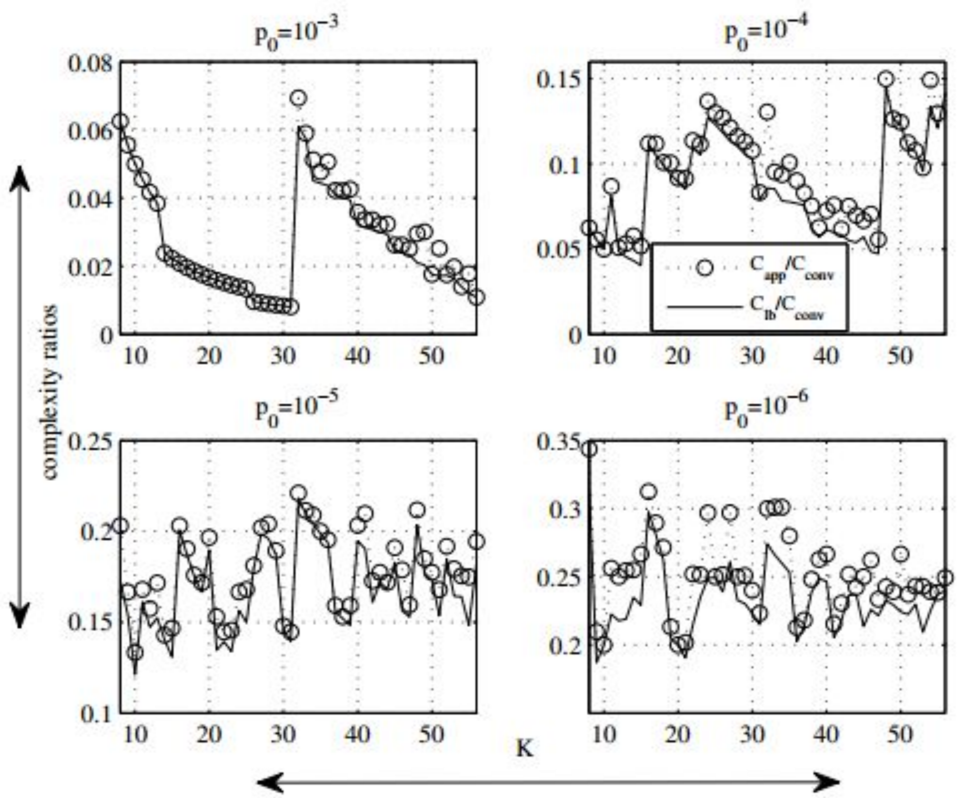

Fig. 7. Complexity ratios versus number of users at Γ0 =15 dB

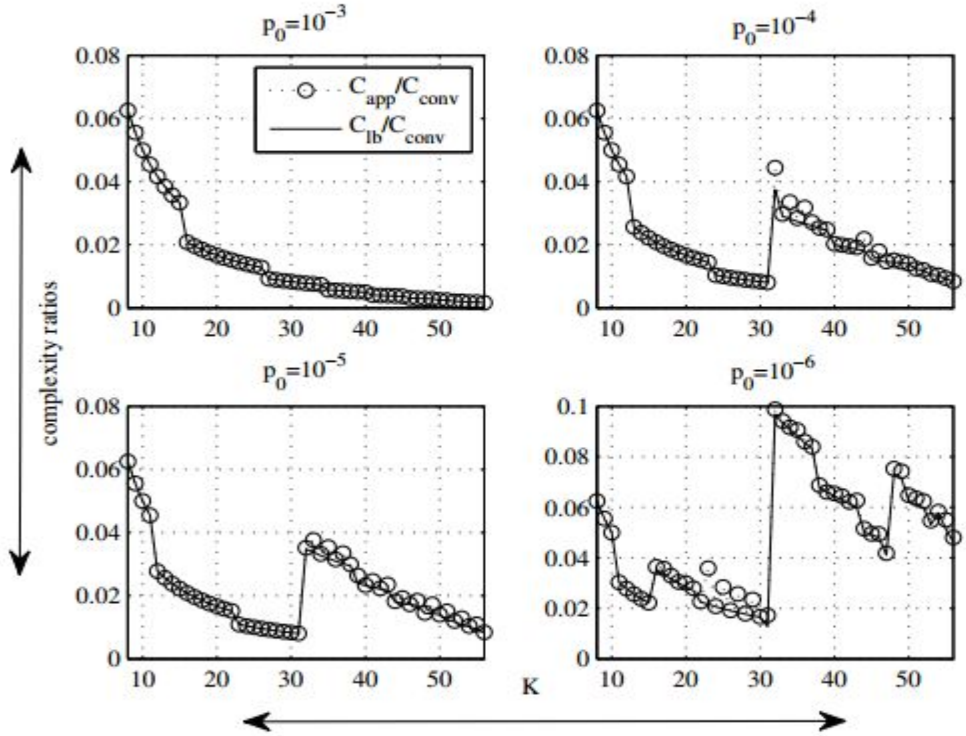

Fig. 8. Complexity ratios versus number of users at Γ0 =25 dB.

VI. CONCLUSIONS

In this paper we introduced the G-OFDM-IDMA concept, which can markedly reduce the complexity of conventional OFDM-IDMA while preserving the BER performance and bandwidth efficiency. Based on the SNR-variance evolution technique, we proposed a grouping algorithm which finds the optimal user grouping profile that has the lowest decoding complexity for given target BER, average SNR and bandwidth efficiency. The grouping problem was formulated as an ILP problem whose sub-optimal solution was given with low optimization complexity. Simulations showed that the sub-optimal solution is close to the lower bound and can markedly reduce the decoding complexity compared to the conventional system. The framework of the proposed grouping algorithm can also be applied to other system optimization problems such as power minimization with decoding complexity constraint.

REFERENCES

1. *L. Ping, L. Liu, K. Wu, and W. K. Leung, "Interleave-division multipleaccess," IEEE Trans. Wireless Commun., vol. 5, no. 4, pp. 938–947, Apr.2006.*
2. *L. Liu, J. Tong, and L. Ping, "Analysis and optimization of CDMA systems with chip-level interleavers," IEEE J. Sel. Areas Commun.,vol. 24, no. 1, pp. 141–150, Jan. 2006.*
3. *K. Li, X. Wang, and L. Ping, "Analysis and optimization of interleavedivision multiple-access communication systems," IEEE Trans. Wireless Commun., vol. 6, no. 5, pp. 1973–1983, May 2007.*
4. *B. Cristea, D. Roviras, and B. Escrig, "Turbo receivers for interleavedivision multiple-access systems," IEEE Trans. Commun., vol. 57, no. 7, pp. 2090–2097, July 2009.*
5. *K. Kusume, G. Bauch, and W. Utschick, "IDMA vs. CDMA: detectors, performance and complexity," in Proc. 2009 Intl. Conf. on Communications, pp. 1–8.*
6. *P. Wang, J. Xiao, and L. Ping, "Comparison of orthogonal and nonorthogonal approaches to future wireless cellular systems," IEEE Veh.Technol. Mag., vol. 1, no. 3, pp. 4–11, Sept. 2006.*
7. *M. Lau and W. Yue, "A linear zero-one formulation of optimal power allocation of IDMA systems," in Proc. 2007 Intl. Conf. on Wireless Communications, Networking and Mobile Computing, pp. 637–640.*

8. Z. Rosberg, "Optimal transmitter power control in interleave division multiple access (IDMA) spread spectrum uplink channels," IEEE Trans. Wireless Commun., vol. 6, no. 1, pp. 192–201, 2007.
9. L. Linton, P. Conder, and M. Faulkner, "Multi-rate communications using layered interleave-division multiple access with power allocation,"in Proc. 2009 Wireless Communications and Networking Conf., pp. 1–5.
10. P. Weitkemper and K.-D. Kammeyer, "Power optimization of IDMA systems with different target BER constraints," in Proc. 2007 IEEE Vehicular Technology Conf. – Spring, pp. 2812–2816.
11. P. Wang, L. Ping, and L. Liu, "Power allocation for multiple access systems with practical coding and iterative multi-user detection," in Proc. 2006 Intl. Conf. on Communications.
12. [12] X. Wang and H. V. Poor, "Iterative (turbo) soft interference cancellation and decoding for coded CDMA," IEEE Trans. Commun., vol. 47, no. 7, pp. 1046–1061, July 1999.
13. [13] L. Ping, "Interleave-division multiple access and chip-by-chip iterative multi-user detection," IEEE Commun. Mag., vol. 43, no. 6, pp. S19–S23, June 2005.
14. I. M. Mahafeno, C. Langlais, and C. J′ ego, "Reduced complexity iterative multi-user detector for IDMA (interleave-division multiple access) system," in Proc. 2006 Global Telecommunications Conf., pp. 1–5.
15. L. Ping, Q. Guo, and J. Tong, "The OFDM-IDMA approach to wireless communications," IEEE Wireless Commun. Systems,vol.14,no.3,pp. 19–24, June 2007.
16. R. Zhang, L. Xu, S. Chen, and L. Hanzo, "EXIT-chart-aided hybrid multiuser detector for multicarrier interleave-division multiple access," IEEE Trans. Veh. Technol., vol. 59, no. 3, pp. 1563–1567, Mar. 2010.
17. Y. Tu, P. Fan, and G. Zhou, "Grouped interleave-division multiple access," in Proc. 2006 Intl. Conf. on Communications and Networking in China, pp. 1–5.
18. H. Bie and Z. Bie, "A hybrid multiple access scheme: OFDMA-IDMA," in Proc. 2006 Intl. Conf. on Communications and Networking in China, pp. 1–3.
19. J. Dang, L. Yang, and Z. Zhang, "On grouped OFDM-IDMA," in Proc. 2011 Asilomar Conference on Signals, Systems, and Computers,pp. 1298–1303.
20. D. E. Knuth, "Big Omicron and big Omega and big Theta," ACM SIGACT News, vol. 8, no. 2, pp. 18–24, Apr.-June 1976.
21. C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: turbo-codes," IEEE Trans. Commun., vol. 44, no. 10, pp. 1261–1271, Oct. 1996.
22. S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, "A soft-input soft-output APP module for iterative decoding of concatenated codes," IEEE Commun. Lett., vol. 1, no. 1, pp. 22–24, Jan. 1997.
23. J. Tong, Q. Guo, and L. Ping, "Analysis and design of OFDM-IDMA systems," Eur. Trans. Telecommun., vol. 19, no. 5, pp. 561–569, Aug. 2008.
24. I. M. Mahafeno, C. Langlais, and C. Jego, "OFDM-IDMA versus IDMA with ISI cancellation for quasistatic Rayleigh fading multipath channels," in Proc. 2006 International Symposium on Turbo Codes,pp. 1–6.
25. A. M. Tonello and R. Bernardini, "Analysis of the achievable timefrequency diversity gains in coded OFDM," in Proc. 2003 Intl. Symp.on Wireless Personal Multimedia Communications, pp. 193–197.
26. L. Ping and L. Liu, "Analysis and design of IDMA systems based on SNR evolution and power allocation," in Proc. 2004 Vehicular Technology Conf., vol. 2, pp. 1068–1072.
27. J. Tong, Q. Guo, and L. Ping, "Performance analysis of OFDM-IDMA systems with peak-power limitation," in Proc. 2008 IEEE Intl. Symp.on Spread Spectrum Techniques and Applications, pp. 555–559.
28. D. Bertsimas and R. Weismantel, Optimization over Integers. Dynamic Ideas, 2005.
29. L. Linton, P. Conder, and M. Faulkner, "Multiple-access communications for underwater acoustic sensor networks using OFDM-IDMA," in Proc. 2009 OCEANS, MTS/IEEE Biloxi - Marine Technology for Our Future: Global and Local Challenges, pp. 1–8.
30. L. Lihai, "Interleave division multiple access," Ph.D. dissertation, City University of Hong Kong, 2005, pp. 100.
31. J. G. Proakis, Digital Communications, 4th edition. McGraw-Hill, 1999, pp. 41–46.
32. D. Tse and P. Viswanath, Fundamentals of Wireless Communication.
33. Cambridge University Press, 2005, pp. 60–64.
34. T. Yang, J. Yuan, and Z. Shi, "Rate optimization for IDMA systems with iterative joint multi-user decoding," IEEE Trans. Wireless Commun.,vol. 8, no. 3, pp. 1148–1153, Mar. 2009.